This assessed homework is intended to remind you how to do calculations in vector calculus (you may wish to go over your MAS165 notes or consult a textbook on vector calculus), since it is an *essential* tool for this module. This is why it is due in early: **Friday 10 October, 4pm**. The homework is worth 15 marks.

1. Let a vector field $\mathbf{A}$ take the following form in cylindrical coordinates:

$$\mathbf{A}(r,\phi,z) = \frac{1}{r^2}\,\hat{\mathbf{r}} + \frac{\cos\phi}{r}\,\hat{\mathbf{z}}.$$

   (a) Express the unit vectors $\hat{\mathbf{r}}$, $\hat{\phi}$ and $\hat{\mathbf{z}}$ in Cartesian coordinates. Show your reasoning in your answer. [2]

   (b) Calculate the surface integral

$$\oint_S \mathbf{A}\cdot d\mathbf{S},$$

   where S is the surface of a cylinder of outer radius R_2 and inner radius R_1 centred around the origin and extending from $z=0$ to $z=L$. [3]

   (c) Calculate $\mathbf{B} = \nabla\times\mathbf{A}$. [3]

   (d) Show that the surface integral

$$\oint_S \mathbf{B}\cdot d\mathbf{S} = 0,$$

   where S is the surface of the same cylinder. [2]

2. A time-varying scalar field is given by

$$V(x,y,x,t) = \frac{1}{3}xt^2 - \frac{1}{2}yzt + z^2t.$$

   (a) Calculate $\partial_t V$. [2]

   (b) Let x and y have the following time dependence:

$$x(t) = at + b \qquad \text{and} \qquad y(t) = \frac{c}{t} \qquad \text{and} \qquad z(t) = d,$$

   with a, b, c and d constants. Calculate $\dfrac{dV}{dt}$. [3]

Please write clearly and show how you obtained your answers. Illegible work will not be marked!